Perturbation-based coupling of Monte Carlo and Burn-up for multiple burnable regions

Paul Cosgrove, Eugene Shwageraus 08/11/17
Contents

• Introduction: Burn-up Methods
• Introduction: Collision History Method
• Previous Work
• Extension to multiple burnable regions
• Implementation of perturbation-based depletion
• Results: Accuracy & Stability
Burn-up

- Time evolution of fuel composition in a reactor
- Euler, Predictor-Corrector, Substep, Stochastic Implicit Euler (SIE)...

\[ \frac{d\vec{N}}{dt} = \overline{A}(\sigma, \phi)\vec{N}(t) \]

- Different strengths and weaknesses…
- Want a **stable** and **accurate** method with minimum expense
Sensitivity Calculations in Serpent

- Collision history method
- Can be used to generate $\frac{\partial \sigma_i}{\partial \Sigma_{t,j}}$

Aufiero, M et al. (2015). “A collision history-based approach to sensitivity/perturbation calculations in the continuous energy Monte Carlo code SERPENT”. In: Annals of Nuclear Energy 85 pp. 245-258
Perturbation-based Depletion: Previous work

\[ S_x^R = \frac{\partial R / R}{\partial x / x} = \frac{\partial \sigma_i}{\partial \Sigma_{t,j}} \times \frac{\Sigma_{t,j,0}}{\sigma_{i,0}} = \frac{\partial \sigma_i}{\partial N_j} \times \frac{N_{j,0}}{\sigma_{i,0}} \]

Can calculate partial derivatives of 1-group XS’s to any nuclide density
Perturbation-based Depletion: Previous work

\[ \sigma_x(\vec{N}) = \sigma_x(\vec{N}_0) + \frac{\partial \sigma_x}{\partial \vec{N}} \cdot [\vec{N} - \vec{N}_0] \]

Can create low-order approximation for how XS’s change with burn-up

Time (or burn-up) dependence of Gd-157 XS

Extension to multiple burnable regions

- Single region was proof of principle
- Depletion problems are necessarily multi-region – Gadolinium pins require fine spatial discretisation for physically accurate analysis
- As well as accuracy in nuclide density, multiple regions raise concerns over spatial stability
Extension to multiple burnable regions

- XS’s in one region now sensitive to perturbations in all other regions
- Requires an additional flux dependence

\[ \phi(\vec{N}) = \phi(\vec{N}_0) + \frac{\partial \phi}{\partial \vec{N}} \cdot [\vec{N} - \vec{N}_0] \]

- Serpent 2.1.29 more efficiently tallies large numbers of sensitivities across multiple regions – previously impossible
Implementation of burn-up scheme

Previously took sensitivity to every perturbation – no longer practical!
Implementation of burn-up scheme

- Need some degree of adaptivity – choose which responses and perturbations are worth including!
- Also must account for uncertainty: zero any $S^R_x$ with uncertainty > 10%
Implementation of burn-up scheme

- Modify Serpent to extract burn-up information for external script
- Requires 2 transport solutions per BOS/EOS (Serpent quirk)
- Substep procedure: LE/QI or LE/LI with perturbations

\[
\sigma_x(t) = \frac{t - t_1}{t_0 - t_1} \sigma_x(t_0) \left( 1 + \frac{\partial \sigma_x(t_0)}{\partial \bar{N}} \cdot [\bar{N} - \bar{N}_0] \right) + \frac{t - t_0}{t_1 - t_0} \sigma_x(t_1) \left( 1 + \frac{\partial \sigma_x(t_1)}{\partial \bar{N}} \cdot [\bar{N} - \bar{N}_1] \right)
\]

- Regular substep for XS’s which do not have sensitivity coefficients

Results: Mini-assembly

- 2D, 3x3 mini-assembly with reflective boundaries
- Contains radially divided Gd-bearing pin
- 2.5% Gd-155 by nuclide density
- Burned for 100 days by Pred-Corr, Substep and perturbation scheme
- Reference LE/QI solution
Results: Mini-assembly

- Good accuracy in predicting Gd-155
- Not so good in other elements – outclassed by Substep
- Runtime a factor of 17x slower on final burn-up step (neglecting cost of second MC solution)
Results: Mini-assembly

- Briefly examined the effect of varying the uncertainty threshold from 5% to 20%
- 10% appears to be roughly optimal for agreement with benchmark

![Graph showing % difference in Gd-155 nuclide density across radial regions with different uncertainty levels.]

![Graph showing % difference in Pu-239 nuclide density across radial regions with different uncertainty levels.]

Results: PWR pin

- 3D, 3.66m tall PWR pin with radially reflective and axially vacuum BCs
- Divided into 20 regions with identical properties and moderator density
- Examine spatial stability with burnup – burn for 310 days with 20 day steps
Results: PWR pin

- Plot Xe-135 density over time from perturbation scheme to inspect stability
Results: PWR pin

- Plot Xe-135 density over time from perturbation scheme to inspect stability
Results: PWR pin

- Plot Xe-135 density over time from perturbation scheme to inspect stability
Results: PWR pin

- Plot Xe-135 density over time from perturbation scheme to inspect stability
Results: PWR pin

- Plot Xe-135 density over time from perturbation scheme to inspect stability
Results: PWR pin

Pu, U densities remain mostly symmetric due to ‘sloshing’ between Predictor and Corrector.
Results: PWR pin

• Comparison against 5 day timestep SIE at 310 days

• Unfortunately, the perturbation scheme appears only conditionally stable!
Results: PWR pin

- Comparison against 20 day timestep Substep at 310 days
- Appears to be some improvement over Substep
Thank you!

Questions? Suggestions?