Introduction to Sensitivity and Uncertainty Analysis in Reactor Physics

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Outline

- Sensitivity analysis
- Uncertainty analysis
- Methods
- Application to reactor physics
- Example calculation

Sensitivity

- *Starting point*: mathematical model containing uncertain parameters and response dependent on this model
- *Question*: If one of the parameters is perturbed, how will this affect the response?
- *Mathematical definition*:
  - Simplest case: local sensitivity of response R with respect to parameter α at point $\alpha = \alpha^0$ is the derivative

$$s_\alpha = \left(\frac{dR}{d\alpha} \right)_{\alpha=\alpha^0} \tag{1}$$

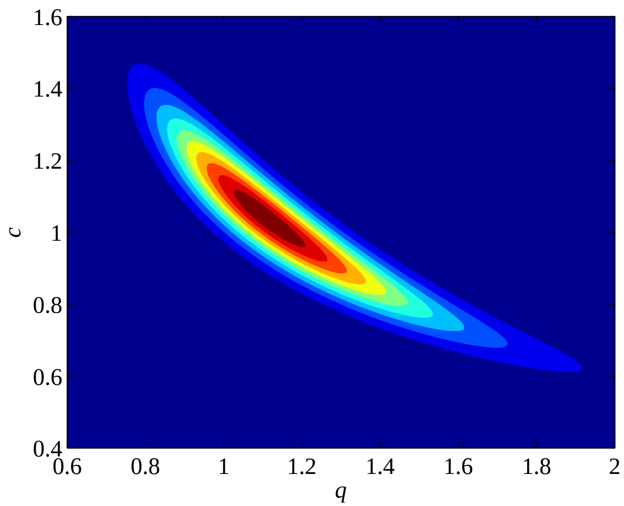
  - This generalizes easily to more general mathematical systems (e.g. parameters that are functions and responses that are functionals)

Sensitivity Analysis

- *Objective:* Compute derivatives with respect to all parameters of interest
- Brute-force approach:
  - Variate the parameters one-by-one and compute the response
  - Inefficient when there are several parameters
- Deterministic approach:
  - Formulate the problem mathematically and compute the derivatives
  - Very efficient if a mathematical concept called *adjoint* is utilized

Uncertainty

- *Starting point:* a mathematical model containing uncertain parameters and response dependent on this model
- *Question:* How to quantify the uncertainty related to the parameters?
  - Bayesian probability definition: knowledge about a parameter presented as probability distribution
  - Variance (one parameter) or covariance (several parameters) of the distribution may be chosen as the descriptive statistic for the uncertainty



Uncertainty Analysis

- *Objective:* Compute the probability distribution of the response based on the probability distributions of the uncertain parameters
- Determination of the exact distribution usually extremely difficult

  $\Rightarrow$ Compute only variance/covariance due to uncertain parameters OR estimate distribution based on simulations
- Inaccuracy related to numerical methods or approximation errors not included in classical uncertainty analysis

Uncertainty Analysis Methods

- Deterministic approach:
  1. Calculate response sensitivity vector $\boldsymbol{s}$
  2. Linearize response
  $$R \approx \boldsymbol{s}\boldsymbol{\alpha} \tag{2}$$
  3. Compute respective variance/covariance
  $$\mathrm{Cov}[R] \approx \mathrm{Cov}[\boldsymbol{s}\boldsymbol{\alpha}] = \boldsymbol{s}\mathrm{Cov}[\boldsymbol{\alpha}]\boldsymbol{s}^T\ . \tag{3}$$
- Statistical approach
  1. Sample points from distribution $p(\boldsymbol{\alpha})$
  2. Compute R corresponding to each sample
  3. Compute uncertainty estimates based on simulated $p(R)$

Application to Reactor Physics

- *Mathematical model*: transport (or diffusion) equation, potentially combined with a depletion model
- *Responses:* multiplication factor, reaction rates, homogenized cross-sections etc.
- *Uncertain parameters:* neutron cross-sections, initial nuclide concentrations, system dimensions etc.

Application to Reactor Physics: Adjoint-based Approach

+ Computationally very efficient
+ Yields detailed sensitivity profiles
– Best-suited for deterministic codes
– Requires extensive modifications in the code
– Has not been applied to depletion problems

Application to Reactor Physics: Statistical Approach

+ Well-suited for both deterministic and Monte Carlo codes
+ Code can be treated as a black box (depletion does not cause any difficulties!)
+ Yields additional information about the distribution $p(R)$ (besides variance/covariance)
– Computationally expensive
– Does not yield sensitivity information

S&U analysis with Monte Carlo method

- Statistical approach
  - Sample from Gaussian distribution based on covariance data
  - Total Monte Carlo:
    - A.J. Koning and D. Rochman, *Towards sustainable nuclear energy: Putting nuclear physics to work, Ann. Nucl. Energy*, **35**, 11, 2024–2030 (2008)
  - Suitable for burnup calculations
- Adjoint-based approach
  - exploit the physical interpretation of adjoint:
    - Brian C. Kiedrowski, Forrest B. Brown and Paul P. H. Wilson, *Adjoint-Weighted Tallies for k-Eigenvalue Calculations with Continuous-Energy Monte Carlo, Nucl. Sci. Eng.*, **168**, 3, 226–141 (2011)
  - Suitable for problems covered by generalized perturbation theory

Example of S&U Calculation

- Calculation code: CASMO-4
- Source of uncertainty: neutron cross-sections
- S&U analysis method: Adjoint-based
- Test case: a 7×7 BWR assembly [1]

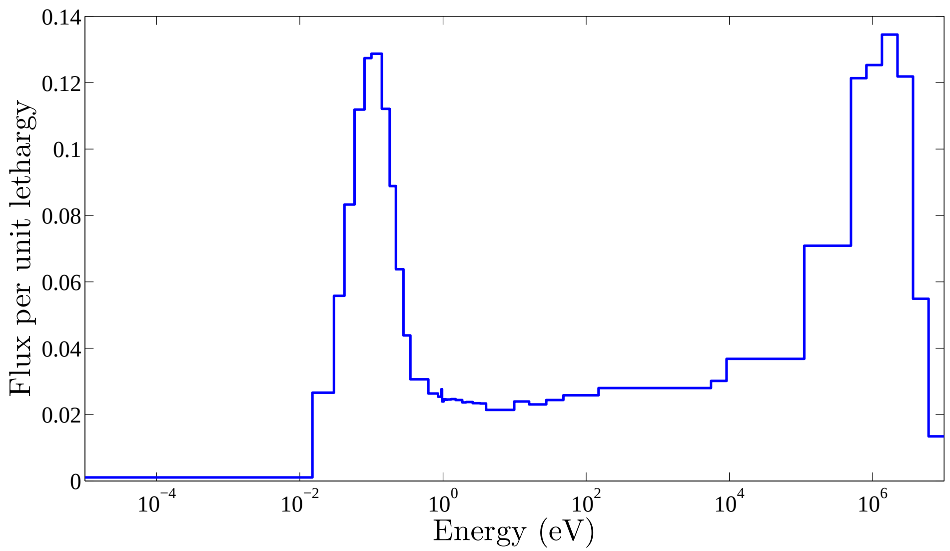
Rod type	^{235}U (wt.%)	Gd_2O_3 (wt.%)	No. of rods
1	2.93	0	26
2	1.94	0	12
3	1.69	0	6
4	1.33	0	1
5A	2.93	3.0	3
6B	2.93	3.0	1

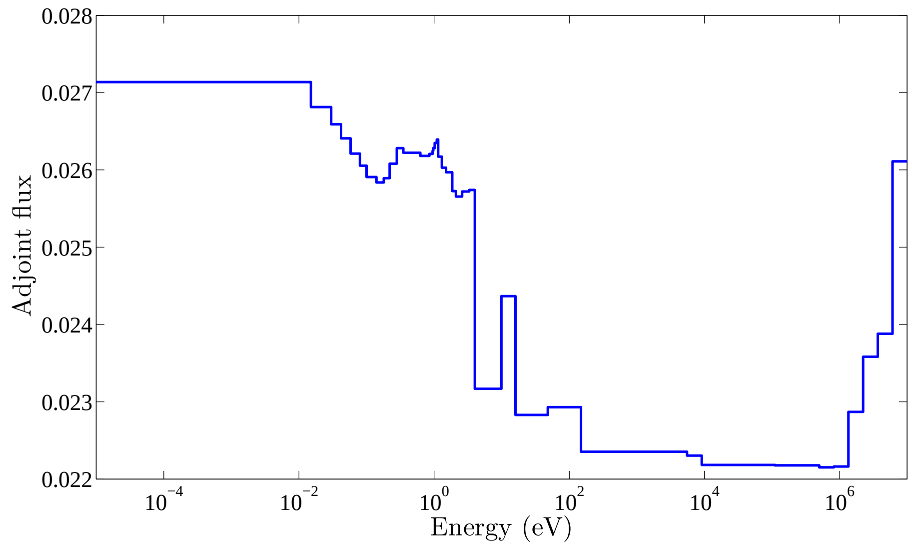
WIDE-WIDE CORNER

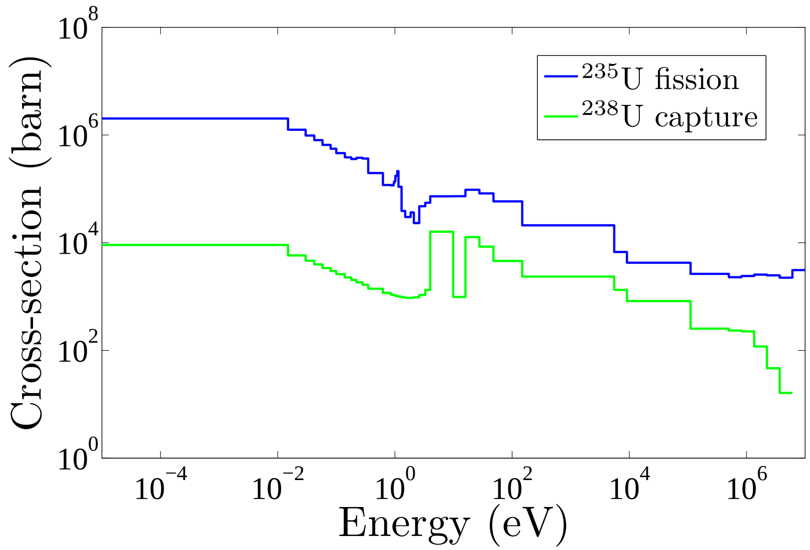
4	3	3	2	2	2	3
3	2	1	1	1	1	2
3	1	5A	1	1	5A	1
2	1	1	1	1	1	1
2	1	1	1	6B	1	1
2	1	5A	1	1	1	2
3	2	1	1	1	2	2

K. Ivanov et al., *Benchmark for uncertainty analysis in modeling (UAM) for design, operation, and safety analysis of LWRs, Volume I: Specification and Support Data for the Neutronics Cases (Phase I)*, Version 2.0 , NEA/NSC/DOC(2011)

Example: flux and adjoint flux







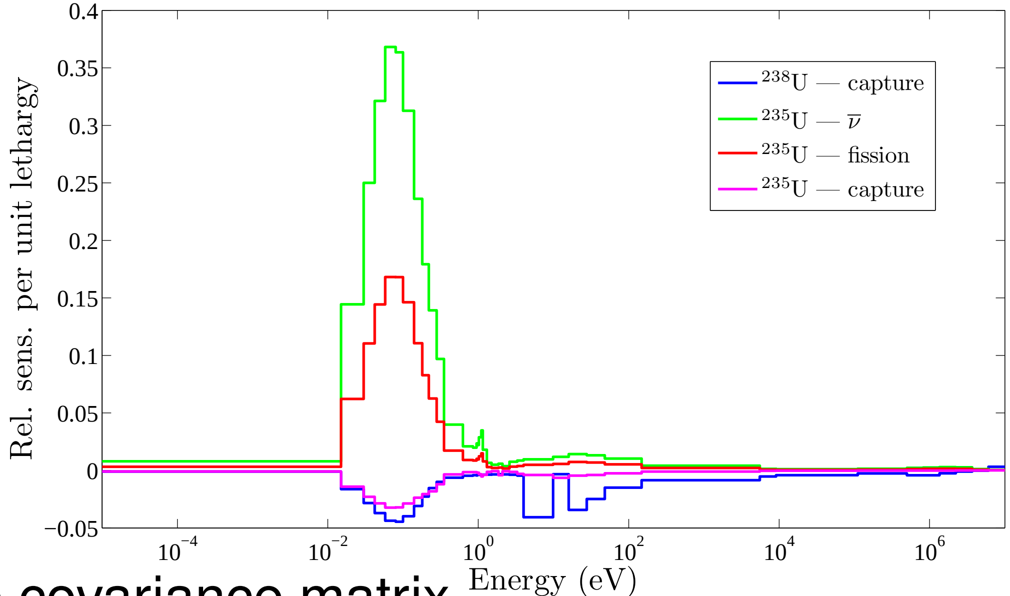
Example: k_{inf} S&U profiles (1)

- $k_{\text{inf}} = 1.1055$
- $\Delta k_{\text{inf}} / k_{\text{inf}} = 0.5076\%$

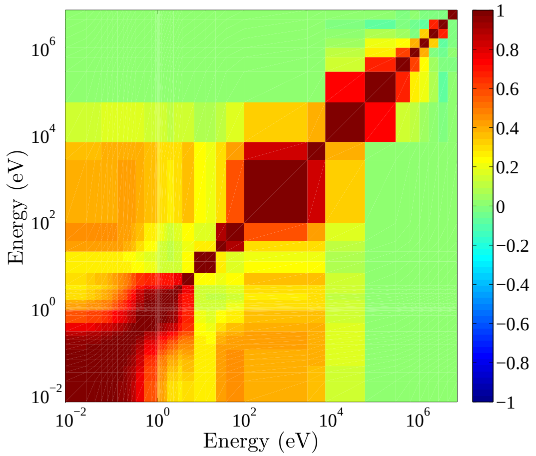
Nuclide	Param.pair	Rel. sensitivity	Rel. uncertainty
^{238}U	σ_c , σ_c	-2.448×10^{-1}	3.198×10^{-1}
^{235}U	ν , ν	9.161×10^{-1}	2.720×10^{-1}
^{235}U	σ_c , σ_c	-1.010×10^{-1}	1.423×10^{-1}
^{235}U	σ_f , σ_f	4.157×10^{-1}	1.416×10^{-1}
^{238}U	σ_s , σ_s	-1.499×10^{-2}	1.320×10^{-1}
^{235}U	σ_c , σ_f		1.242×10^{-1}
^{235}U	χ , χ	9.161×10^{-1}	1.030×10^{-1}
^{238}U	ν , ν	6.107×10^{-2}	7.102×10^{-2}
^{1}H	σ_c , σ_c	-1.072×10^{-1}	5.362×10^{-2}
^{1}H	σ_s , σ_s	1.263×10^{-1}	5.061×10^{-2}

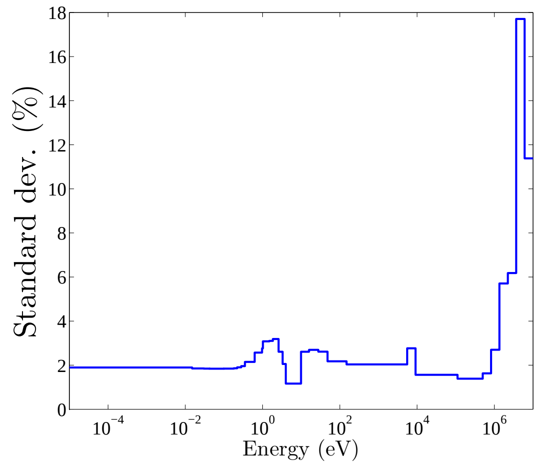
Example: k_{inf} S&U profiles (2)

- Sensitivity plots



- ^{238}U capture covariance matrix

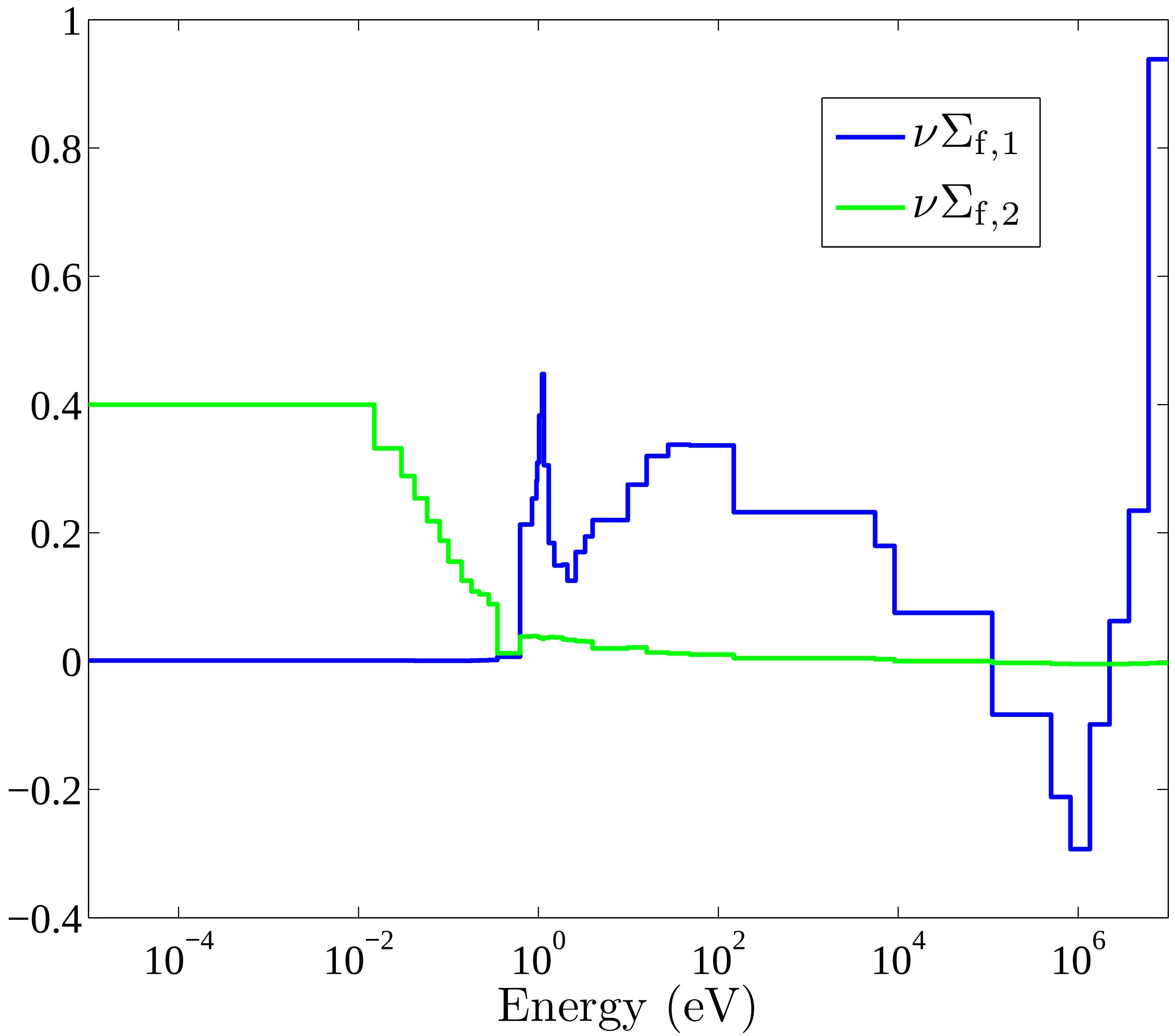




Example: Homogenized two-group cross-section uncertainties

Response R	Value	Relative uncertainty $\frac{\Delta R}{R}$ (%)
$\nu\Sigma_{\mathrm{f},1}$	4.976×10^{-3}	8.399×10^{-1}
$\nu\Sigma_{\mathrm{f},2}$	6.922×10^{-2}	4.490×10^{-1}
$\Sigma_{\mathrm{a},1}$	7.283×10^{-3}	7.526×10^{-1}
$\Sigma_{\mathrm{a},2}$	5.494×10^{-2}	2.122×10^{-1}
$\Sigma_{\mathrm{c},1}$	5.348×10^{-3}	1.098×10^{0}
$\Sigma_{\mathrm{c},2}$	2.653×10^{-2}	5.066×10^{-1}
$\Sigma_{\mathrm{f},1}$	1.935×10^{-3}	5.563×10^{-1}
$\Sigma_{\mathrm{f},2}$	2.841×10^{-2}	3.244×10^{-1}

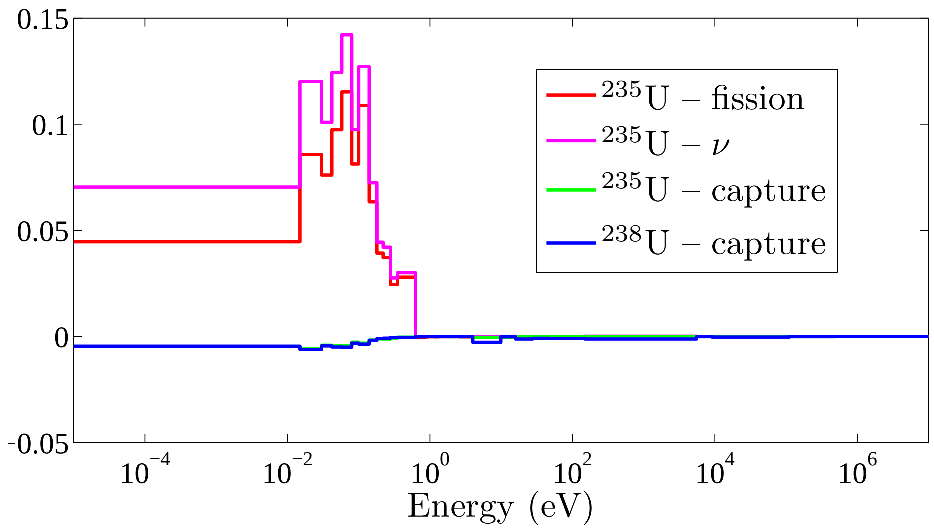
Example: generalized adjoints



Example: S&U profiles for $\nu\Sigma_{\mathrm{f},2}$

- $\nu\Sigma_{\mathrm{f},2} = 6.922 \times 10^{-2}$, relative uncertainty 4.490×10^{-1}%

Nuclide	Param. pair	Sensitivity	Contribution to $\frac{\Delta R}{R}$ (%)
^{235}U	$\overline{\nu}, \overline{\nu}$	9.996×10^{-1}	3.105×10^{-1}
^{235}U	$\sigma_\mathrm{f}, \sigma_\mathrm{f}$	7.985×10^{-1}	2.893×10^{-1}
^{235}U	$\sigma_\mathrm{f}, \sigma_\mathrm{c}$	7.985×10^{-1}	1.134×10^{-1}
^{238}U	$\sigma_\mathrm{c}, \sigma_\mathrm{c}$	-4.406×10^{-2}	7.257×10^{-2}
^{235}U	$\sigma_\mathrm{c}, \sigma_\mathrm{c}$	-3.599×10^{-2}	5.613×10^{-2}



Summary

- Sensitivity analysis
  - Adjoint-based approach
  - Brute force method
- Uncertainty analysis
  - Deterministic (requires sensitivities)
  - Statistical sampling
- S&U analysis with Monte Carlo method
  - Statistical sampling based on covariance data
  - Total Monte Carlo
  - Adjoint-based (exploit physical interpretation of adjoint)